3D Geometry

In Cartesian geometry, the three coordinate axes, **X**, **Y** and **Z**, are all perpendicular to each other and intersect at the origin. Like all points in 3D space assigned with an ordered triple of real numbers, the origin is given by **O** (0,0,0). Each number gives the distance of the point from the origin when measured along any given axis. This is equal to the distance of the point from the plane formed by the other two axes. The coordinate axes are the sets of points:

{(x,0,0)} (**x-axis**), {(0,y,0)} (**y-axis**), {(0,0,z)} **(z-axis**)

X, **Y** and **Z** are the positive directions of the axes, while **X'**, **Y'** and **Z'** are the negative directions.

The coordinate planes are the sets of points:

{(x,y,0)} (**xy-plane**), {(0,y,z)} (**yz-plane**), {(x,0,z)} (**xz-plane**)

These planes divide 3D space into eight octants:

The positive octant **XOYZ** consists of points (x,y,z), where x,y,z > 0.

The negative octant **X'OY'Z'** consists of points (x,y,z) where x,y,z < 0.

The other six octants are **X'OYZ, XOY'Z, X'OY'Z, XOYZ', X'OYZ'** and **XOY'Z'.**

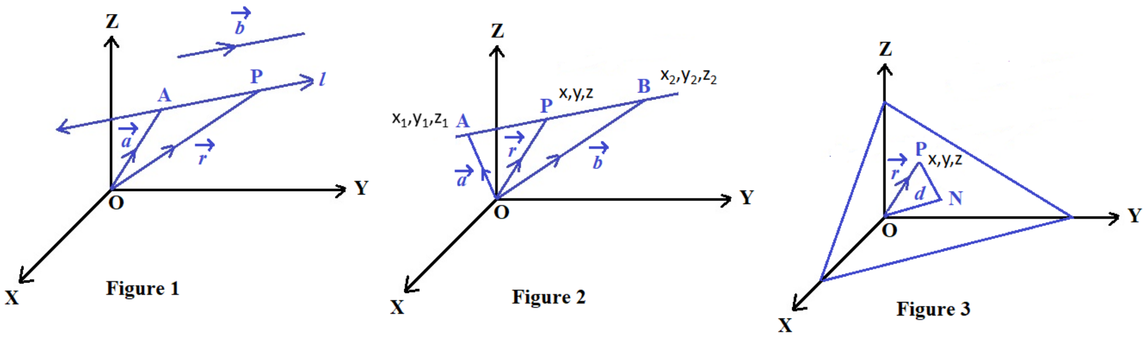


Figure 1: A line **l** passing through point **A** and parallel to a given vector $\vec{b}$ in 3D space is of the form, $\vec{r} = \vec{a} + \lambda\, \vec{b}$, where $\vec{r}$ is the position vector of any point **P** on the line, $\vec{a}$ is the position vector of **A** with respect to the origin **O**, and **λ** is some real number.

Figure 2: Thus, given two points **A** (x_1,y_1,z_1) and **B** (x_2,y_2,z_2) on a line, position vectors $\vec{a}$ and $\vec{b}$ of **A** and **B**, respectively, $\vec{r}$ position vector of a point **P** (x,y,z), **P** lies on the line IF

$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$. This equation can be described in Cartesian form as:

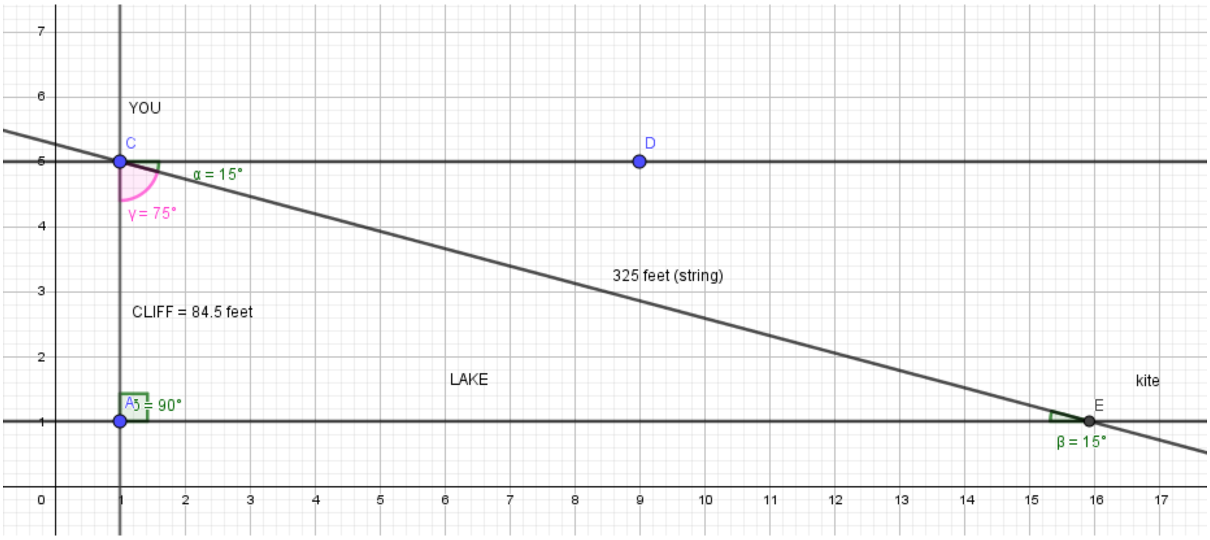
$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Figure 3: A plane containing point **P**, where $\underset{n}{\rightarrow}$ is the unit normal vector along the normal from the origin to the plane, and d is the perpendicular distance from the origin, is described by the vector form of its equation given by $\underset{r}{\rightarrow}.\,\hat{n}$ = d.

Cubes, prisms, pyramids, spheres, cones and cylinders are all 3-D objects; they can be rotated in space. A **solid** can be obtained by rotating a plane curve about a fixed line as its axis. The plane curve is the **generatrix** of the surface. Thus, if the generatrix and axis are parallel, the surface you get is a **cylinder**. If the generatrix is a line and it intersects the axis line, you get a **right circular cone** with its apex at the point of intersection. Curves and lines of intersection of solids are common features in engineering applications. For instance, **conic sections** are curves obtained at the intersection of cones and planes. **Parabola, ellipse and hyperbola are all types of conic sections**. A **circle** is a special case of an **ellipse**. Parabola, ellipse and circle can be obtained by intersection of one cone with a plane at different angles to the axis of the cone. A hyperbola is obtained upon the intersection of a double-napped cone with a plane.

Helices are interesting features seen in many molecules in Nature: double helix in DNA, a-helix in proteins, helix seen in virus capsid protein, triple helix in collagen protein. For instance, in DNA, the staggered stacking of the nucleotides that takes advantage of the 3'→5' phosphodiester bonds between the sugars of successive nucleotides along each strand and the hydrogen bonding between the bases of the two strands confers a helical nature to the molecule. A right-handed helix is traced by a point **(x(t),y(t),z(t))**; its pitch is 2π, slope is 1, radius is 1 about the z-axis, and **x(t) = cos(t); y(t) = sin(t); z(t) = t** (or some multiple of t).

Hint for Assignment:



You (kite flier is C), lake is AE, E is kite, CE is string (325 feet). Line of sight (CD), angle of declination is 15°, so angle ACE is 75° as ACD is 90°. Angle DCE = angle CEA = 15°.

Which trigonometric ratio of which angle will give you the height of the cliff?

$f(x)=x^5-7x^4+9x^3+23x^2-50x+24$, peak in 1st quadrant in XY plane

This is what your solid of rotation should look like: